APPLICATION OF ADVANCED STATISTICAL METHODS IN ASSESSMENT OF THE LATE PHASE OF A NUCLEAR ACCIDENT

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The paper presents a new methodology for improving of estimates of radiological situation on terrain in the late phase of a nuclear accident. Methods of Bayesian filtering are applied to the problem. The estimates are based on combination of modeled and measured data provided by responsible authorities. Exploiting information on uncertainty of both the data sources, we are able to produce improved estimate of the true situation on terrain. We also attempt to account for model error, which is unknown and plays crucial role in accuracy of the estimates. The main contribution of this paper is application of an approach based on advanced statistical methods, which allows for estimating of model error covariance structure upon measurements. Model error is estimated on basis of measured-minus-observed residuals evaluated upon measured and modeled values. The methodology is demonstrated on a sample scenario with simulated measurements.

A) INTRODUCTION

In case of an accident in a nuclear power plant, there could be an aerial release of radioactive pollutants into the living environment. If such a release occurs, there is a radioactive plume moving over the terrain. During this phase, the plume cause primarily exposure from the cloud and internal exposure due to inhalation. Due to deposition processes, the plume is depleted during passing over terrain and leaves a radioactive trace on the ground. The time interval spanning from the time of the release start to time when the radioactive cloud leaves area of interest (in meso-scale modeling up to few tens or hundreds kilometers from the source) is called the plume phase.

After the plume phase, post-emergency phase (late phase) follows. It covers latter stages of accident consequences evolution. Post-emergency phase may extend over a prolonged period of several weeks or many years depending on the source of radiation and local conditions. It ends when environmental radiation levels resume to normal. The main exposure pathways in this phase are external exposure due to radionuclides deposited on the ground, internal exposure due to inhalation of resuspended material (in some cases) and also internal exposure due to ingestion of contaminated foodstuffs as the deposited material migrates through the root system to the edible parts of crops consumed by people and livestock [1].

This paper deals with assessment of radiation situation in the late phase of an accident. Knowledge of spatio-temporal distribution of radioactive material in this phase is essential for emergency management and planning of late phase countermeasures. The distribution of radioactive material on terrain can be modeled via models taking into account both the radioactive decay and also removing of material caused by environmental processes.

We are aware of the fact that model results are not perfect and error can be in this field of modeling high (tens or even hundreds of percents compared to the real magnitude). The main sources of uncertainties are in model input parameters (source term, weather forecast, etc.), in wrong conceptualization of the physical problem and there are also inherited uncertainties caused by stochastic nature of the problem. The only information of the true situation on terrain is provided by measurements, which are assumed to be more accurate than model predictions. Adjustment of model predictions in a way to be in accordance with these measurements can increase their reliability. The process of combining information provided by mathematical model and measurements is referred as data assimilation.

B) DATA ASSIMILATION

Bayesian approach to filtering is applicable to all linear and nonlinear stochastic systems. Its principle consists in combining of the information provided by the model with the measured data. Bayesian estimation procedure has two iteratively repeated steps. The first step transits the state estimate to the next time step. From the known state estimate in time t evaluates the prediction of the state in time t+1. This step is called time update. In the second step called data update, the information provided by actual measurements $\mathbf{y}_{t+1}$ is included into the current estimate, which is being adjusted towards these measurements.

Kalman filter is simple implementation of the Bayesian filter and is widely employed in many fields. Its usage is limited to the case of linear estimation with the Gaussian noise. Under these assumptions leads general Bayesian filtering scheme to the Kalma filter equations for time update and data update steps [9]. The equations perform recursive update of the first two moments of estimated Gaussian distribution - the mean value $\mathbf{x}$ and its covariance matrix $\mathbf{P}$. The unavoidable condition for application of Kalman filter is knowledge of model and measurement error.

C) ASSIMILATION SCENARIO

Our assimilation scenario covers the post-emergency phase. The source of pollution is placed into the center of polar network. We perform our calculations on subset of this network in successive time steps. All the calculations are made in terms of groundshine-dose as it can be easily measured and other dosimetric quantities can be calculated from it. Groundshine-dose in ordered set of analyzed spatial points forms our state vector $\mathbf{x}$. We assume $\mathbf{x} \sim N(\mathbf{x^*},\mathbf{P})$, where $N(\mathbf{x},\mathbf{P})$ is multi-dimensional Gaussian distribution with mean value $\mathbf{x}$ and covariance matrix $\mathbf{P}$, $\mathbf{x^*}$ is the state estimate. Let $\mathbf{x^*}_0$ be an initial estimate of groundshine-dose and $\mathbf{P}_0$ its corresponding error covariance matrix. This background-field is given by probabilistic version of Atmospheric Dispersion Model (ADM) and constitutes the prior characterization of the problem. It is based on segmented Gaussian plume model and it is part of the HARP system, more in [8]. We assume sparse measurements $\mathbf{y}$ of actual gamma dose-rate to be available each time step. These measurements are assumed to be conditionally independent with known error.

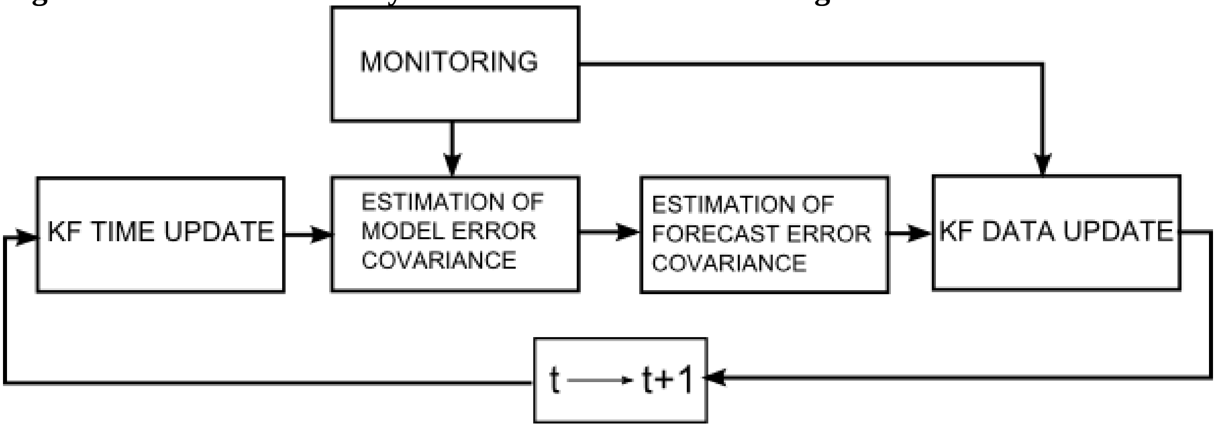
As the problem is treated as linear-Gaussian, it can be solved via Kalman filter. Provided that the model error covariance matrix is known, the time update of forecast error covariance matrix $\mathbf{P}_t$ is calculated as $\mathbf{P}_t = \mathbf{M}\mathbf{P}_{t-1}\mathbf{M}+\mathbf{Q}_t$ [9]. The value of $\mathbf{Q}$ should reflect total (unknown) model error, which is contribution to the forecast error due to differences between the model and the true process in each step. It is obvious that if $\mathbf{Q}$ is neglected, the predicted forecast error will be underestimated. This could cause divergence from the true state (its good estimate) because smaller model error will handicap the information provided by measurements.

1. Estimation of model error covariance matrix

In our case, the model error covariance matrix $\mathbf{Q}$ is unknown and it has to be estimated before application of KF each assimilation step. As the total number of model error covariance elements to be estimated is much higher than the number of measurements, we can't estimate all of them. Simplified covariance model based on idealized assumptions has to be introduced.

Schematically, let the model error covariance matrix be approximated as a function Q(θ) of a parameter vector θ. Function Q has to be chosen properly in order to produce positive semi-definite symmetric matrices, which can be covariance matrices. The covariance parametrization is constructed upon physical background of the problem, more in [3], [10].

Figure 1. Schematic of one cycle of recursive assimilation algorithm.



For finding the most plausible values of θ a similar approach as proposed in [4] based on modeled-minus-observed residuals is used. Instead of maximum likelihood estimates proposed there we use marginalized particle filter described for example in [2]. The marginalized particle filter is a powerful combination of the particle filter and the Kalman filter, which can be used when the underlying model contains a linear sub-structure, which is being subject to Gaussian noise.

When the measurements are available, we can evaluate residual vector **v** having the same dimension as the measurement vector. We assume **v** to be normally distributed with zero mean value and covariance matrix dependent on estimated parameters $\mathbf{v} \sim N(\mathbf{0}, \mathbf{S}(\theta))$. The form of residual covariance is derived in [4]. The most plausible values of parameters are found each time step via particle filter from multiple evaluation of likelihood $p(\mathbf{v}_t|\theta_t)$ for different parameter vectors θ_t from the set $\{\theta_t^{(1)}, \theta_t^{(2)},..., \theta_t^{(N)}\}$. The likelihood is the higher, the higher is the probability that the difference between modeled and measured values is zero given the covariance $\mathbf{S}(\theta_t^{(i)})$. Incorporation of this algorithm into Kalman filter assimilation scheme results in marginalized particle filter for estimation of joint probability density function $p(\mathbf{x}_t, \theta_t| \mathbf{y}_t)$ which is a mixture of Gaussian and nonparametric distributions.

The schematic of assimilation procedure is in the Figure 1. Assimilation procedure consists of two iteratively repeated steps: In time update step current state estimate together with its forecast error covariance matrix are propagated forward in time. The model error is estimated and accounted for. Following data update step produces so called analysis - adjusts the model prediction to be in accordance with actual measurements. Along with this two Kalman filter steps is in each time step estimated model error and added to the forecast error. The algorithm is in detail explained in [10].

D) NUMERICAL EXAMPLE

Among many radionuclides released during emergency situations, we focus only on Cs-137. Its half-time of decay is long (30 years) and also analysis after the Chernobyl accident had shown that it is one of the most significant nuclides in these types of accidents having detrimental long-term effects on population health.

Groundshine-dose evolution is modeled via semi-empirical formulas from Japan model OSCAAR. This abbreviation stands for Off-Site Consequence Analysis code for Atmospheric Releases in reactor accidents. It has been developed within the research activities on probabilistic safety assessment at the Japan Atomic Research Institute [6] and besides radioactive decay it is capable to take into account the decrease of groundshine due to environmental processes, such as radionuclide migration deeper into the soil, weathering, leaching etc.

For experimental demonstration of the algorithm, an artificial scenario with local rain during the fifth hour of the plume phase was chosen. The rain increases depletion of the plume due the wet deposition. The area of interest is subset of polar network comprising of 91 analyzed points. The measurements were simulated via linear forward observation operator where the true initial deposition $\mathbf{x}_0$ was assumed to be two times higher than the prior estimate $\mathbf{x}^*_0$ obtained from ADM. The initial estimate of forecast error covariance matrix was also provided by ADM where the rain intensity was treated as random variable with given probability distribution. This provided us a valuable physical knowledge but this process also introduced strong covariances among states. Before assimilation, the covariances were reduced in a way of element-wise multiplication with matrix generated by the means of second order autoregressive function [3]. Presentation of visualized assimilation results with comments will be given in oral presentation.

E) CONCLUSION

The results are in compliance with our expectations for this special scenario. Model predictions were successfully adjusted in accordance with the measurements correcting the speed of dose mitigation. Even thought it seems that the methodology has a potential for improving of reliability of predictions in the late phase, the algorithm still has to be improved in terms of robustness and carefully tested.

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